A Note on Cordial, Edge Cordial Labeling of Pythagoras Tree Fractal Graphs

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Abstract: This paper deals with the concept of self-similarity fractals of two types of Pythagoras tree symmetric and asymmetric graphs with existence of cordial and Edge cordial labeling. A square graph is considered as base for constructing the Pythagoras tree fractals which leads to construction of both symmetric and asymmetric type fractals. For our study each iteration and the generalized form are considered as a graph. Eventually each graph is checked with cordial and edge cordial and total cordial, total edge cordial labeling.

Keywords: Fractals, Pythagoras tree, cordial, Edge cordial labeling.

1. Introduction.

A Graph G = <V, E, ψ> consists of a non empty set V called the set of nodes (points, vertices) of the graph, E is said to be the set of edges (may be empty) of the graph and ψ is the mapping from the set of edges E to a set of ordered or unordered pair of elements of V. It would be convenient to write a graph G as <V, E> or simply as G.

A graph labeling is an assignment of integers to the vertices or edges, or both subject to certain conditions. Many types of labeling like harmonious, graceful, etc. are used by various researchers[3,4,6] in practice. A graph G with q edges is *harmonious* if there is an injection f from the vertices of G to the group of integers modulo q such that when each edge 'xy' is assigned the label $|f(x) + f(y)|$ (mod q), the resulting edge labels are distinct.

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A graph G with q edges is *graceful* if f is an injection from the vertices of G to the set $f : V \to \{0, 1, \ldots, q\}$ such that, when each edge 'xy' is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct. Eventually after the introduction of the concept of cordial labeling by (I. Cahit, [4]) many researchers have investigated graph families or graphs which admit cordial labeling with minor variations in cordial theme like product cordial labeling, total product cordial labeling and prime cordial labeling (F. Harary [7]). The brief summary of definitions which are useful for the present investigations are given below.

Definition 1.1 If the vertices of the graph are assigned values subject to certain conditions then it is known as *graph labeling*.

For a dynamic survey on graph labeling we refer to (J.A. Gallian, [6]). A detailed study on variety of applications of graph labeling is reported in (G. S. Bloom, [3]).

Definition 1.2 Let G be a graph. A mapping $f: E(G) \to \{0, 1\}$ is called *binary edge labeling* of G and $f(e)$ is called the label of the edge e of G under f.

For an edge $e = uv$, the induced edge labeling $f^*: E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $vf(0)$, $vf(1)$ be the number of vertices of G having labels 0 and 1 respectively under f while $ef(0)$, $ef(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

Definition 1.3 A binary vertex labeling of a graph G is called a *cordial labeling* if $|vf(0) - vf(1)| \leq 1$

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and $|ef\ (0)\ - ef\ (1)| \leq 1$. A graph *G* is *cordial* if it admits cordial labeling.

Definition 1.4 Let *G* be a graph with two or more vertices then the *total graph T(G)* of a graph *G* is the graph whose vertex set is *V(G)* ∪*E(G)* and two vertices are adjacent whenever they are either adjacent or incident in *G*.

Definition 1.5 A binary edge labeling of a graph *G* is called an edge *cordial labeling* if $|vf\ (0)\ - vf(1)| \leq 1$ and $|ef\ (0)\ - ef\ (1)| \leq 1$. A graph *G* is edge *cordial* if it admits cordial labeling

Definition 1.6 Cahit [4] introduced *edge-cordial labeling* as a binary edge labeling $f: E(G) \rightarrow \{0,1\}$, with the induced vertex labeling given by $f(v) = \sum_{uv \in E} f(uv) (mod\ 2)$ for each $v \in V$ such that $|ef\ (0)\ - ef\ (1)| \leq 1$. And $|vf\ (0)\ - vf(1)| \leq 1$, where $ef(i)$ *and* $vf(i)$ (i = 0 ,1) denote the number of edges and vertices labeled with 0 and 1 respectively.

Definition 1.7 As an extension of the above, we define a *total edge-cordial labeling* of a graph G with vertex set V and edge set E as an edge-cordial labeling such that number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differ by at most 1 (i.e) $\left|\left(v_f(0)+e_f(0)\right)-\left(v_f(1)+e_f(1)\right)\right| \leq 1$. A graph with a total edge-cordial labeling is called a *total edge-cordial graph*.

The present work is focused on cordial and edge cordial labeling of two types of Pythagoras tree fractal graphs namely symmetric and asymmetric.

2. Main results

A fractal [2] on all scales is an object or quantity that displays self-similarity in a somewhat technical sense. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales.

Pythagoras Tree is a plane fractal constructed from squares. It is named after Pythagoras, because each triple of touching squares encloses a right triangle, in a conjuration traditionally used to depict the Pythagorean Theorem. The same procedure is then applied recursively to the two smaller squares. By using squares and 45-45-90 triangles, we can create symmetric model Pythagorean tree fractal. Similarly by using a 30-60-90 triangle instead, we can make this tree bend on one side, which creates a lopsided Pythagoras tree or called asymmetric Pythagorean tree. The following figure illustrates the some iterations of construction of both types of Pythagoras trees. For our study every iteration of the Pythagorean tree is considered as graph.

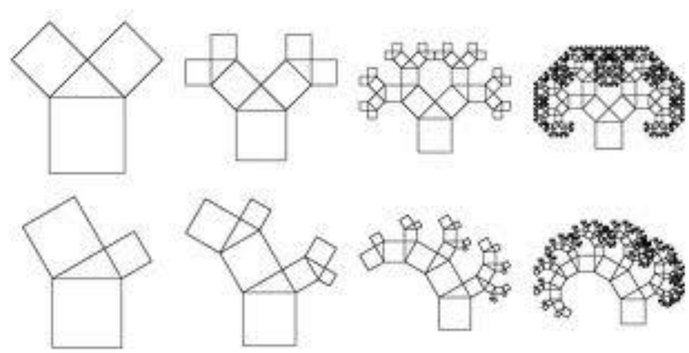

Fig 2.1. Construction of Pythagoras tree.

Pythagorean tree has very wide range applications in Antennas and other similar areas. A novel modified microstrip-fed ultrawide-band (UWB) printed Pythagorean tree fractal monopole antenna is presented by Pourahmadazar J., [8]. In this, by inserting a modified Pythagorean tree fractal in the conventional T-patch, much wider impedance bandwidth and new resonances being produced. By only increasing the tree fractal iterations, new resonances are obtained. The designed antenna has a compact size of 25 × 25 × 1 mm^3 and operates over the frequency band between 2.6 and 11.12 GHz for VSWR <; 2. Using multifractal concept in modified Pythagorean tree fractal antenna design makes monopole antennas flexible in terms of controlling resonances and bandwidth.

A. Aggarwal [1] used a fractal patch antenna using Pythagoras tree as the fractal geometry is presented for dual frequency ultra-wide bandwidth operation. The existence of infinite fractal geometries and their advantages opens the door to endless possibilities to accomplish the task at hand. The use of fractals provides with a bigger set of parameters to control the antenna characteristics. The antenna designed works on 2.4 GHz and 3.5 GHz WiMAX band, which is a next generation internet access network.

2.1 Symmetric model

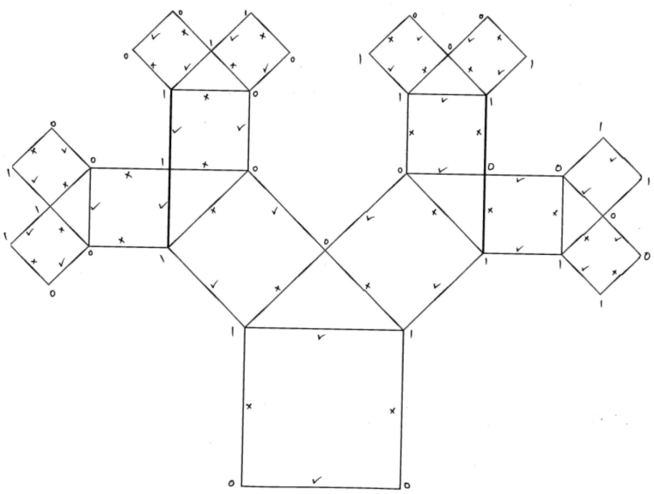

Fig 2.2 cordial labeling (symmetric form)

The initial iteration started with a base square and two more squares are conjoined in 45 degree which creates the first iteration. Likewise the two squares are taken as base square and with angle preservation by applying Pythagorean law the second iteration created with three triangles and seven squares. Similar procedure is adopted for 'n' number of iterations. The initial square is labeled with two 0's and two 1's in vertices to satisfy the condition of cordial labeling. Further each square is preserved vertex labeling in the same fashion without affecting the generality of cordial labeling in every iteration. The law of cordial labeling and total cordial labeling are checked and preserved in every iteration. The above figure (fig 2.2) clearly shows the cordial labeling of Pythagorean tree for third iteration and the same fashion may be continued for 'n' iterations. The vertices are labeled with 0's and 1's and the edges are denoted by tick mark (√) for zeros and ones are denoted by a cross mark (x) to distinguish. The table 2.1 depicts the cordial and total cordial labeling hold good in every iteration.

Similar to previous, the initial square is labeled with two 0's and two 1's in adjacent edges to satisfy the condition of edge cordial labeling. Further each square is preserved edge labeling in the same fashion without affecting the generality of edge cordial labeling in every iteration. The edges are labeled with 0's and two 1's but the vertices are denoted by a tick mark(√) for zeros and by a cross mark (x) for ones to distinguish. The law of edge cordial labeling and total edge cordial labeling are checked and preserved in every iteration. The following figure(fig 2.3) clearly shows the edge cordial labeling of Pythagorean tree for third iteration and the same fashion may be continued for 'n' iterations. The table 2.1 depicts the edge cordial and total edge cordial labeling hold good in every iteration.

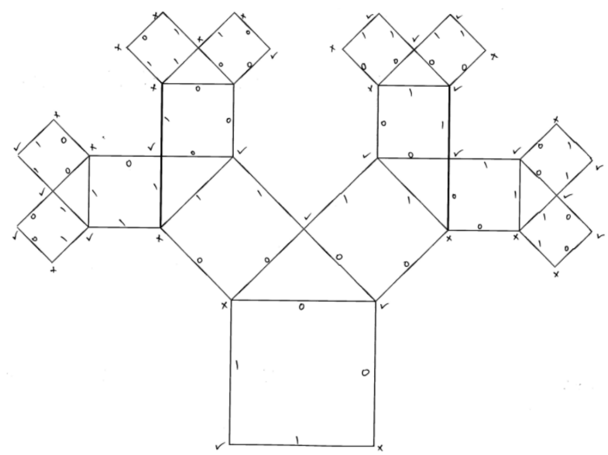

Fig 2.3 Edge cordial labeling (symmetric form)

2.2 Asymmetric model

Asymmetric model is just a change of angle but not in sense. As previous, a base square is considered initially and two more squares are conjoined in 30^0 and 60^0 to satisfy the condition of

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Pythagoras law. The same procedure is adopted to construct the tree for ‘n’ iterations. The asymmetric model is just a flip of symmetric model. Hence, the labeling of vertices to satisfy the condition of cordial and total cordial labeling are preserved as such and the labeling of edges too preserved to satisfy edge cordial and total edge cordial labeling. The table 2.1 clearly depicts the existence of cordial and edge cordial for both symmetric and asymmetric models to hold good in every iteration. The following figures 2.4 and 2.5 shows the cordial and edge cordial labeling of Pythagorean tree of asymmetric model.

Fig 2.4 Cordial labeling (Asymmetric model)

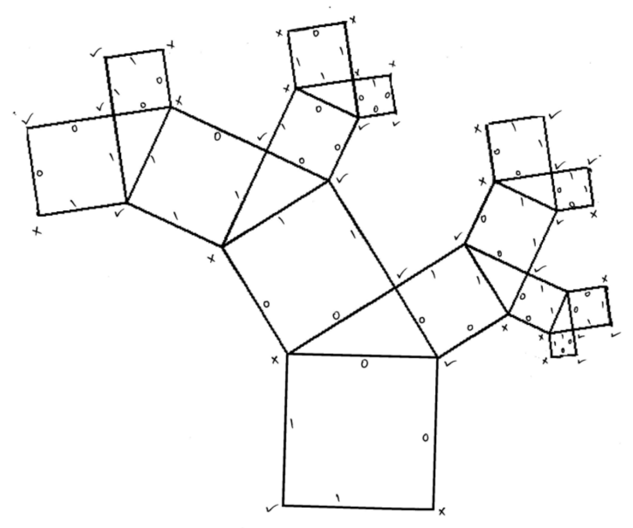

Fig 2.5 Edge cordial labeling (Asymmetric model)

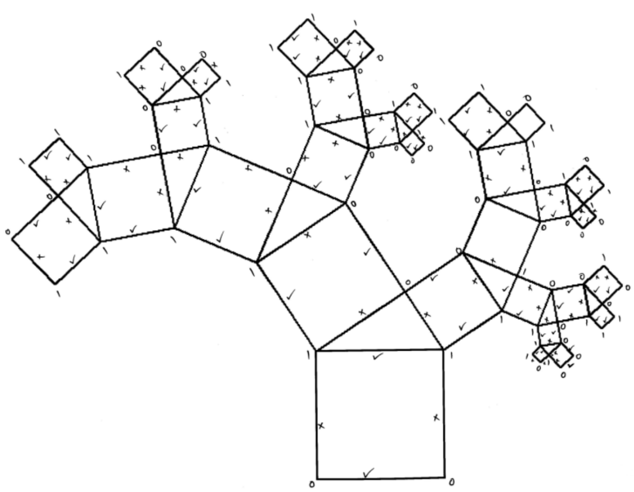

Table 2.1

Iterations	No. of Squares	No. of Triangles	No. of Edges	No. of Vertices
1	3	1	|*ef (0)*|=*6* |*ef (1)*|=*6*	|*vf (0)*|=5 |*vf(1)*|=*4*
2	7	3	|*ef (0)*|=*14* |*ef (1)*|=*14*	|*vf (0)*|=10 |*vf(1)*|=*9*
3	15	7	|*ef (0)*|=*30* |*ef (1)*|=*30*	|*vf (0)*|=20 |*vf(1)*|=*19*

4	31	15	*|ef (0)|=62* *|ef (1)|=62*	*|vf (0)|*=40 *|vf(1)|=39*
N	$2^{n+1}-1$	$2^{n-1}-1$	*|ef (0)|*= $8(2^{n-1})-2$ *|ef (1)|*= $8(2^{n-1})-2$	*|vf (0)|*= $5(2^{n-1})$ *|vf(1)|*=$5(2^{n-1})-1$

3. Conclusion

In this paper we have developed existence of the cordial and edge cordial labeling for both symmetric and asymmetric models of Pythagorean tree fractal graphs. This labeling may lead to some application of the Pythagorean tree in further emerging engineering and science fields. The existence of above said labeling are proved and the results are provided as detailed in table. Hence, it is concluded that the Pythagorean tree is cordial, total cordial, edge cordial and total edge cordial.

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